Modeling the Pollution of Pristine Gas

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With Liubin Pan (CfA) & John Scalo (UT)

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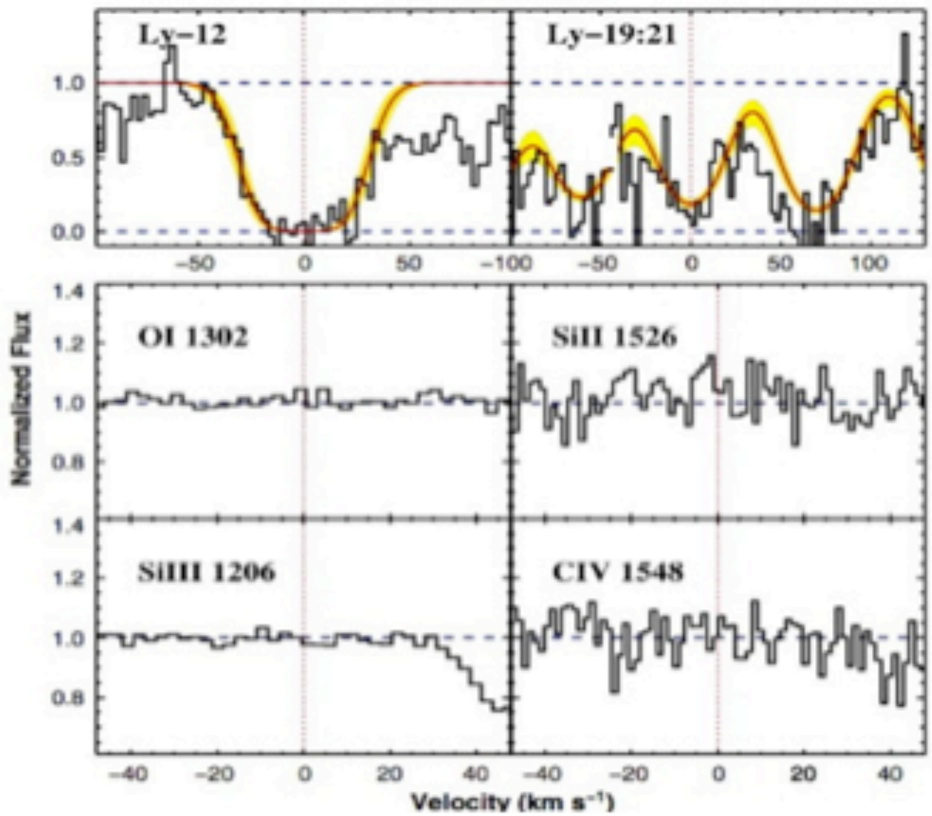
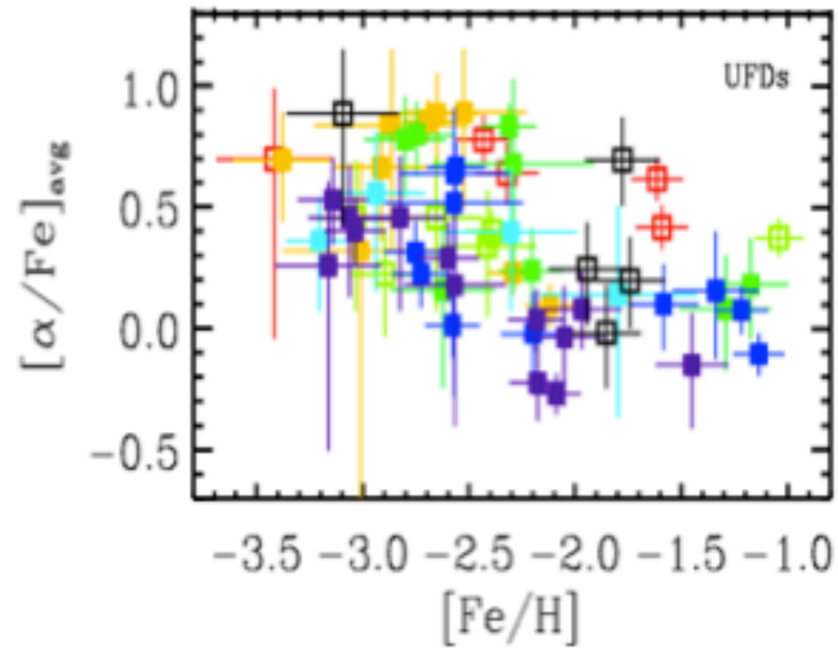
Evolution of the Metal-Free Fraction

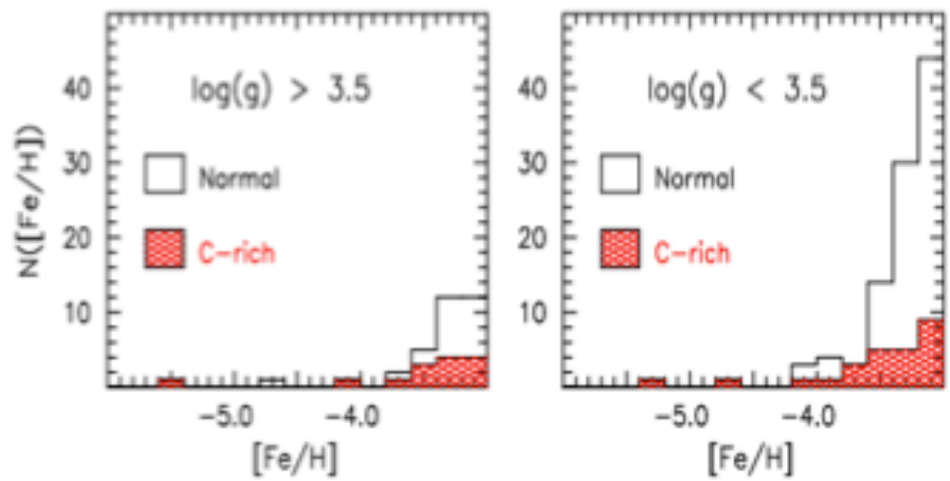


	LLS1134a	LLS0596B
Redshift	3.410883 ± 0.000004	3.096221 ± 0.000009
$\log N_{HI}$	17.95 ± 0.05	17.18 ± 0.04
logD/H	−4.69 ± 0.13*	–
b_{HI} (km s^{-1})	15.4 ± 0.3	20.2 ± 0.8
Temperature (K)	< (1.43 ± 0.05) × 10^4	<(2.48 ± 0.19) × 10^4
Metallicity ($Z_\odot$)	$<10^{-4.2}$	$<10^{-3.8}$
$\log x_{HI}$	≤2.10	≤2.40
$\log n_H$	≤1.86	≤1.98
logU†	≥3	≥3

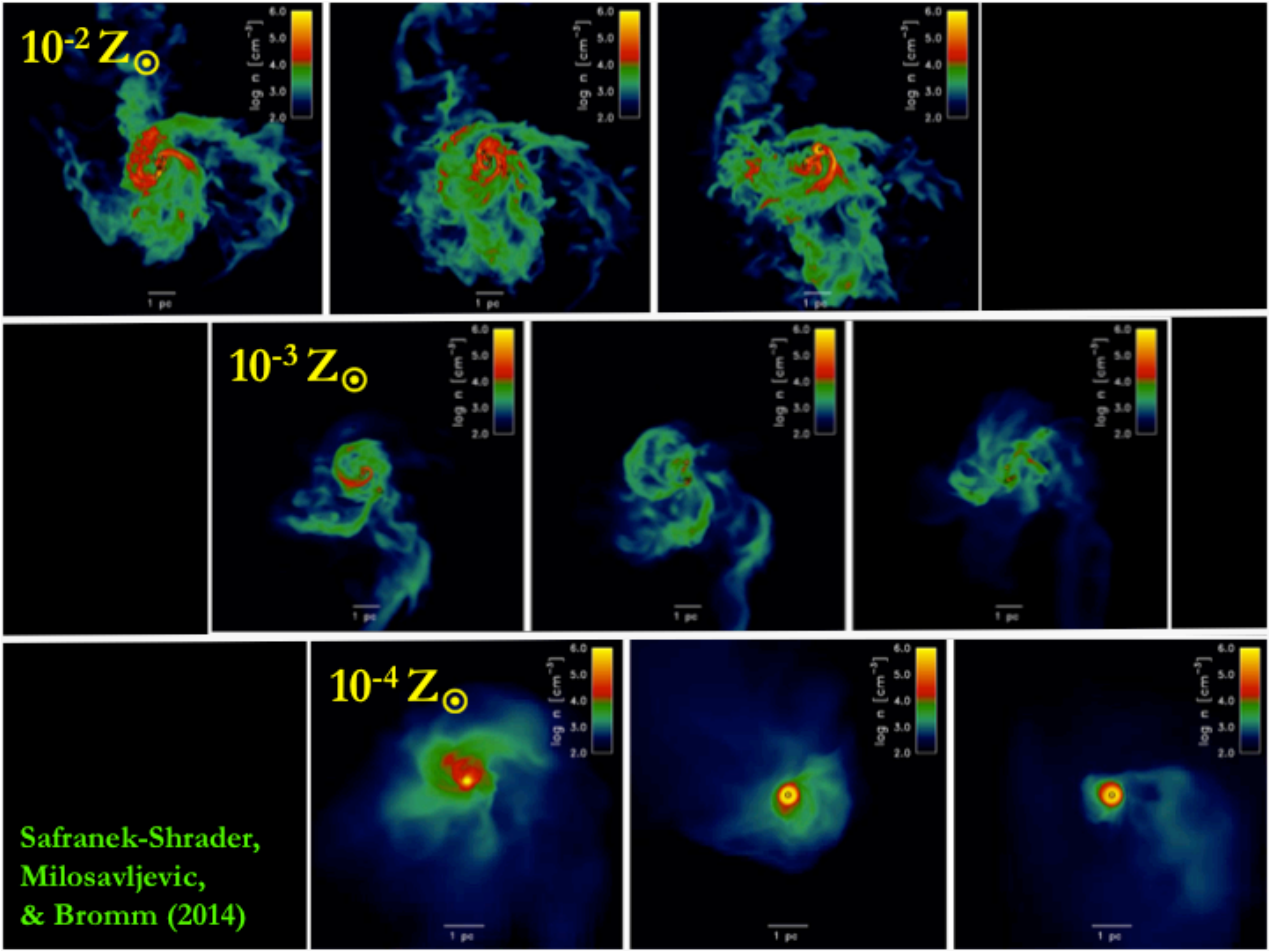
Fumagalli et al (2011) see also Simcoe (2012)



Vargas, Geha, Kirby, & Simon (2013)

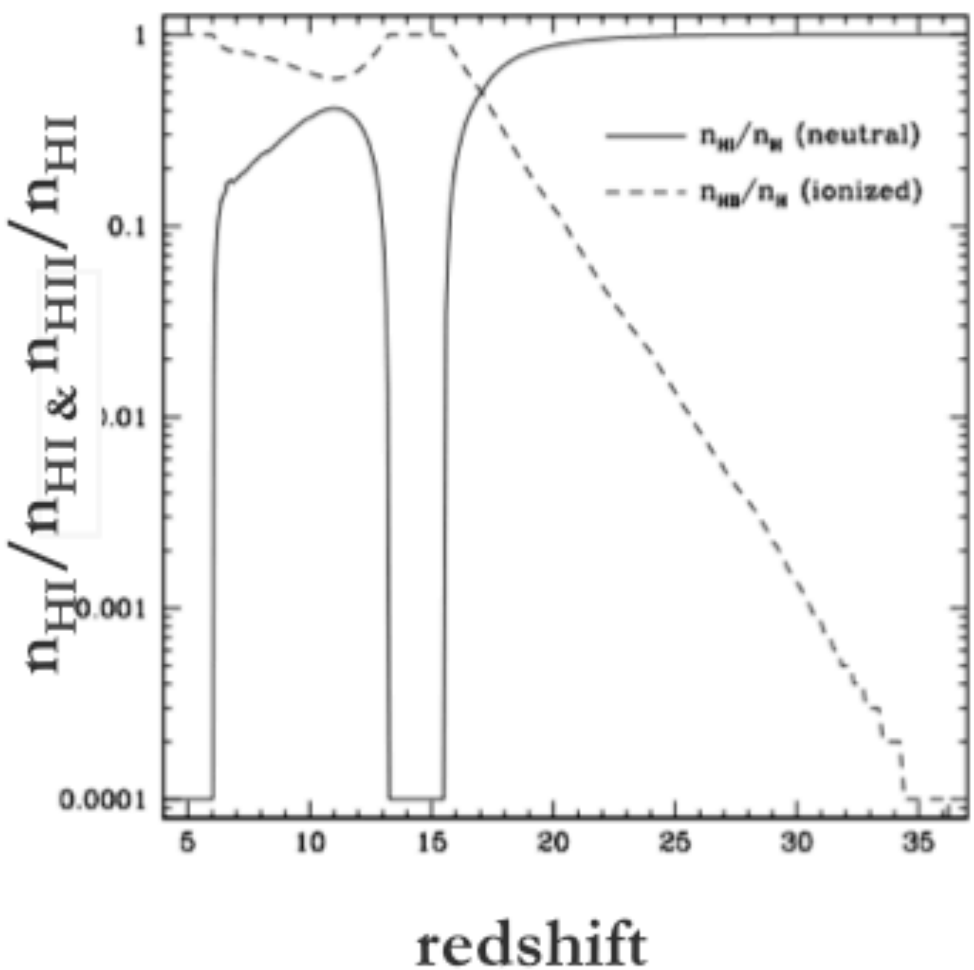


Frebel & Norris (2013)

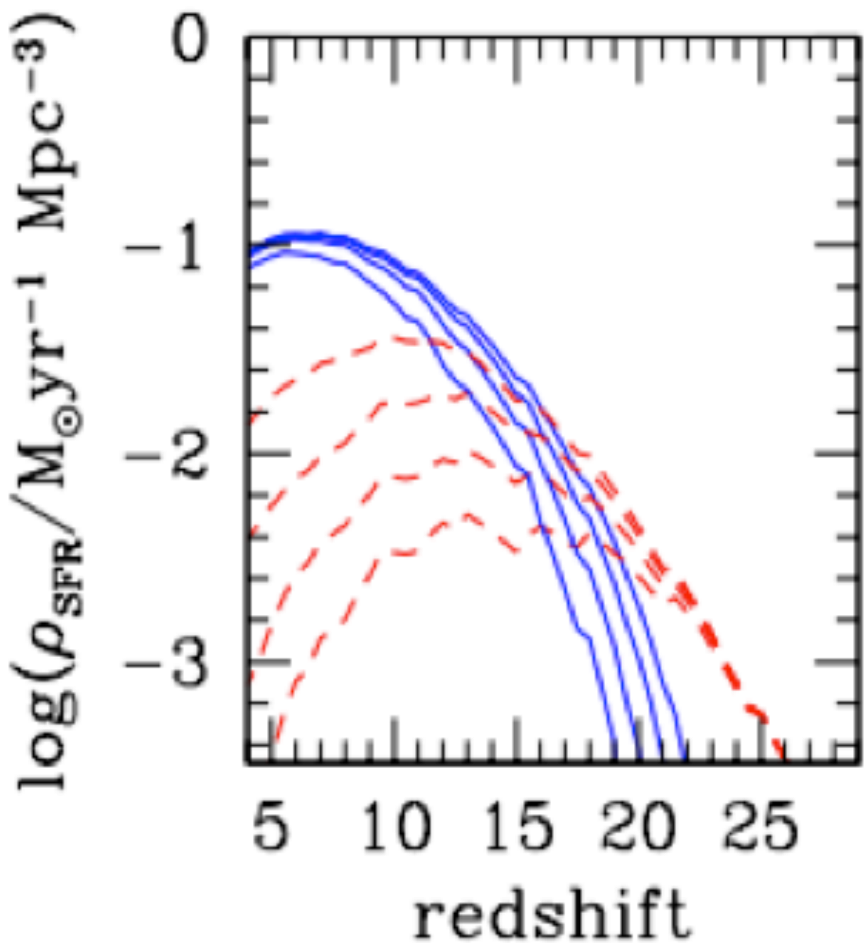


10-2 Z☉
10-3 Z☉
10-4 Z☉
log n [cm-3]
6.0
5.0
4.0
3.0
2.0
1 pc
Safranek-Shrader,
Milosavljevic,
& Bromm (2014)

Population III -> II Transition

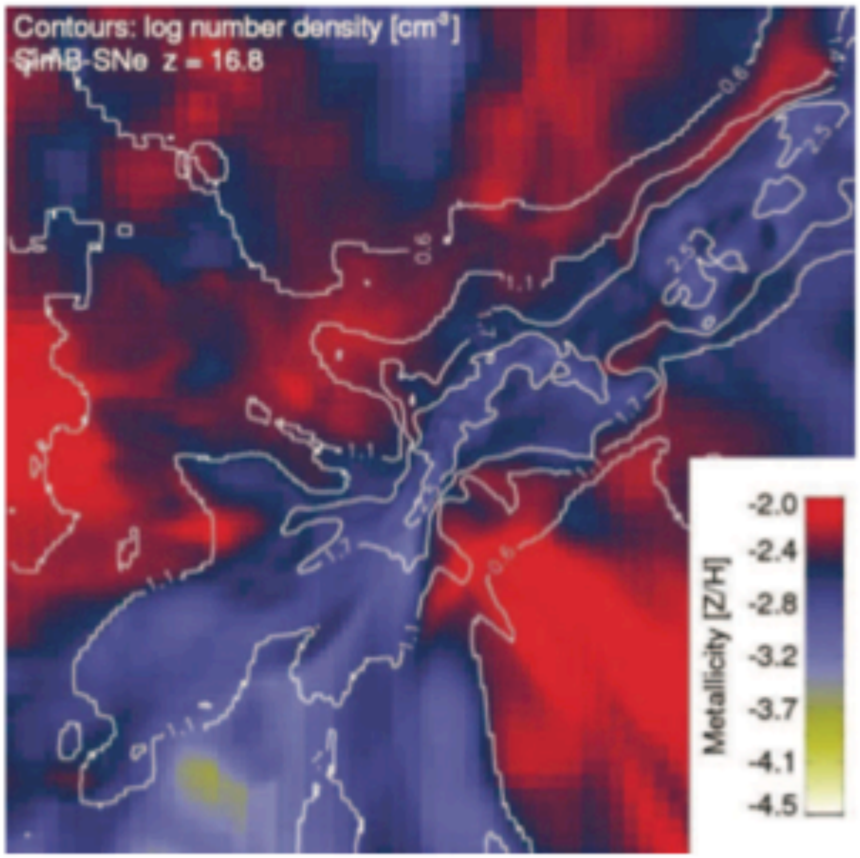


Cen (2003)

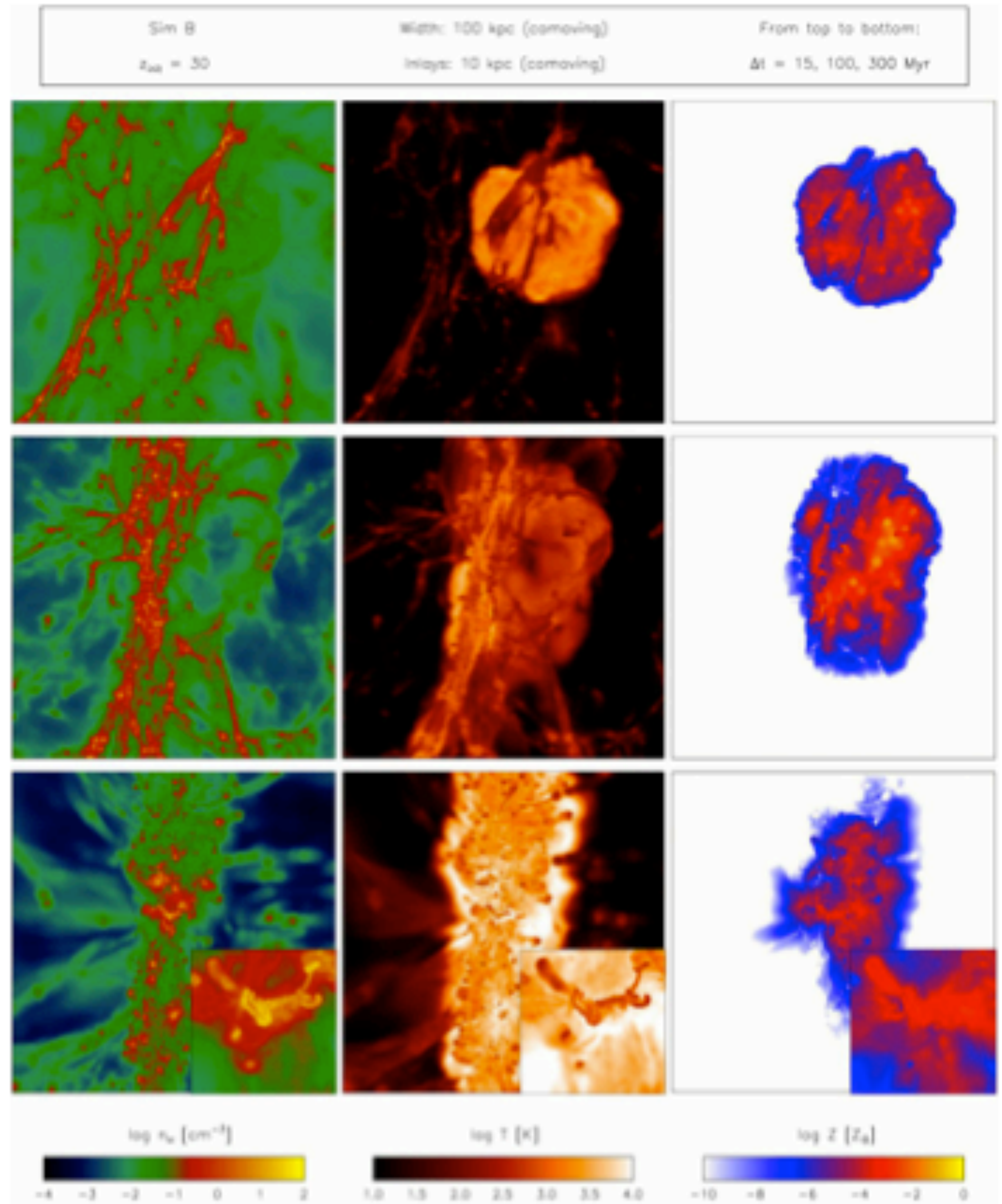


ES, Schneider, & Ferrara (2003)

Population III -> II Transition



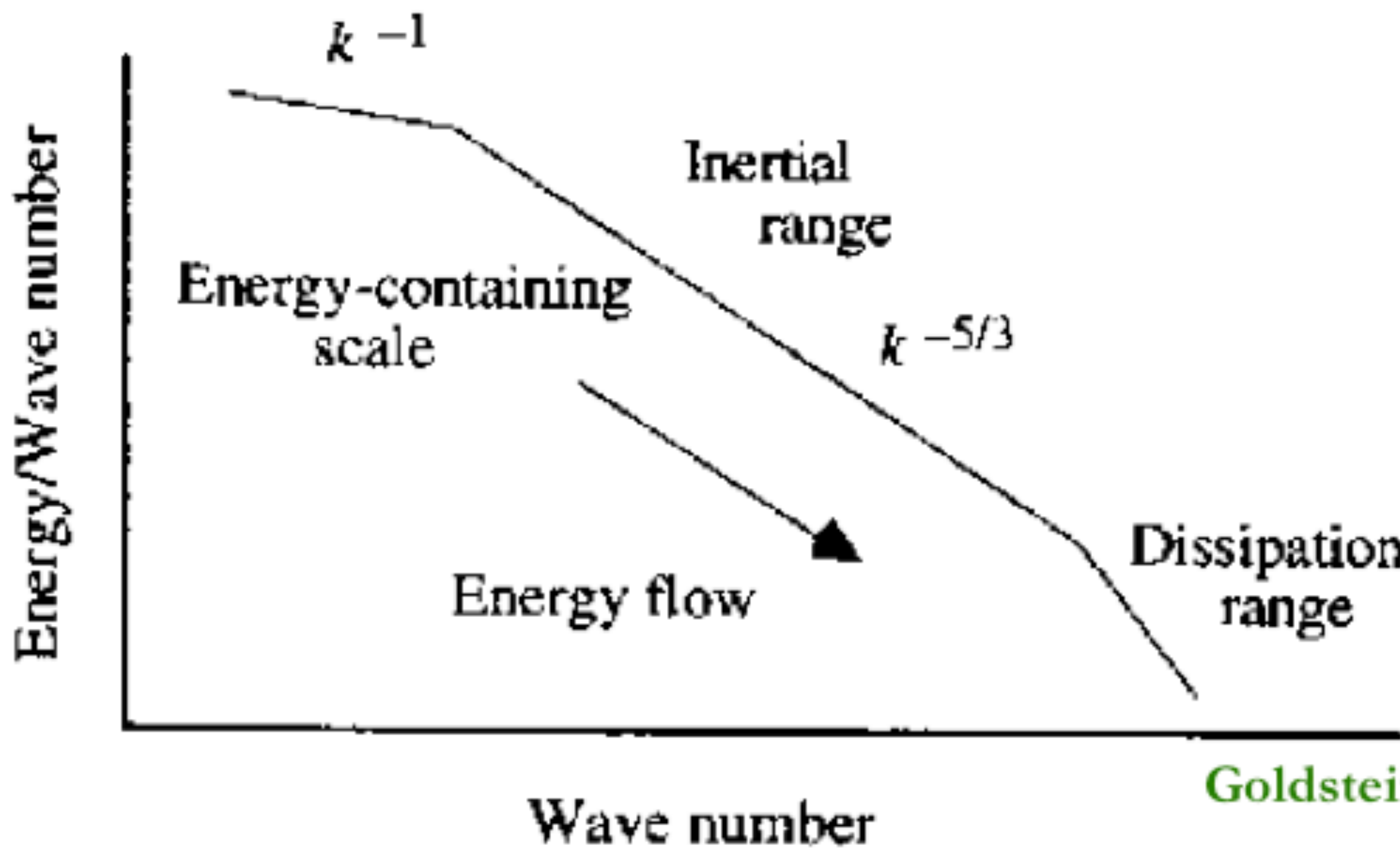
Wise & Abel (2008)



Greif, Glover, Bromm, & Klessen (2010)

See also Ritter, Safranek-Shrader, Gnat, Milosavlkevic, & Bromm (2012)

Mixing is Driven by (Unresolved) Turbulence



Goldstein et al. (1995)

Ionized Medium

Spitzer :

$L \approx 10^{-2}$ pc V_{10} n^{-1} $T_5^{5/2}$

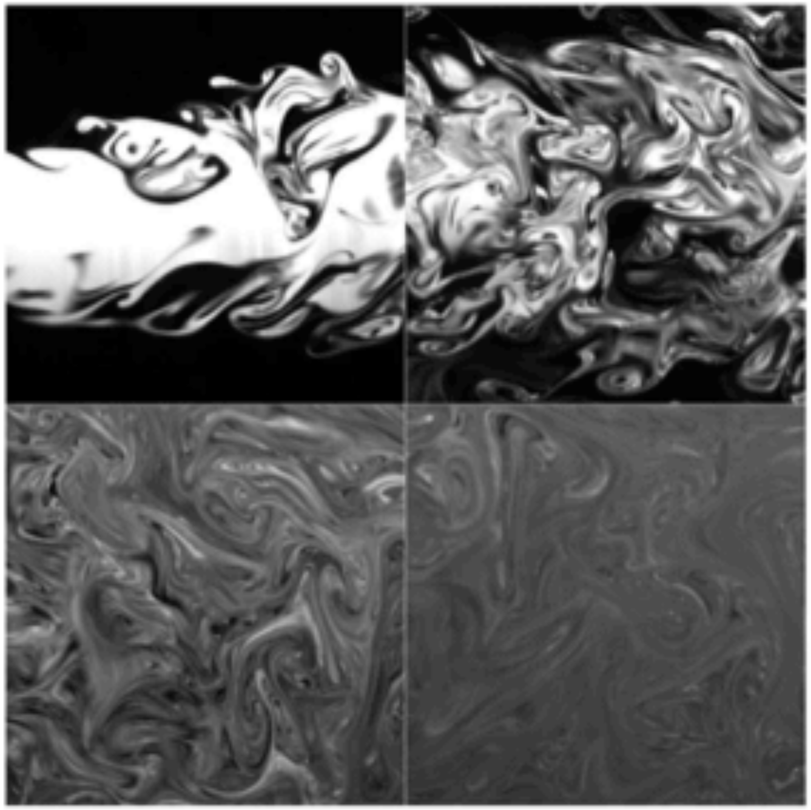
Neutral Medium

Ambipolar Diffusion :

$L \approx 10^{-3}$ pc $V_{A,10}$ n^{-1} $/x_{-6}$

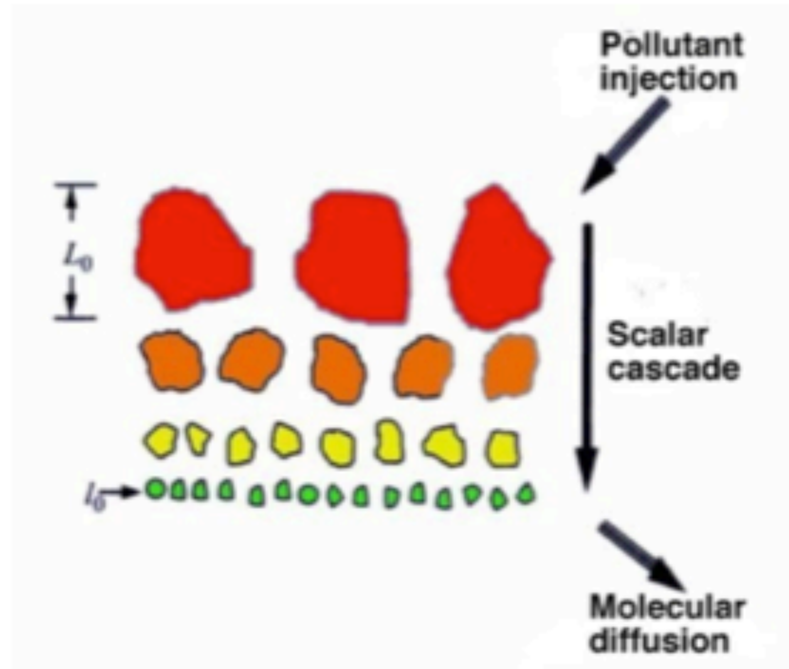
Mixing is Driven by (Unresolved) Turbulence

Turbulent Mixing



Mixing of a jet of dye

Classic phenomenology (Obukohov-Corrsin)



Mixing timescale $\tau_{mix} \sim L/V_t$

Duplat & Villermaux (2008)

Simulations

The FLASH code with modified "Stir unit" for flow and scalar driving

A periodic box of unit size with 512^3 cells.

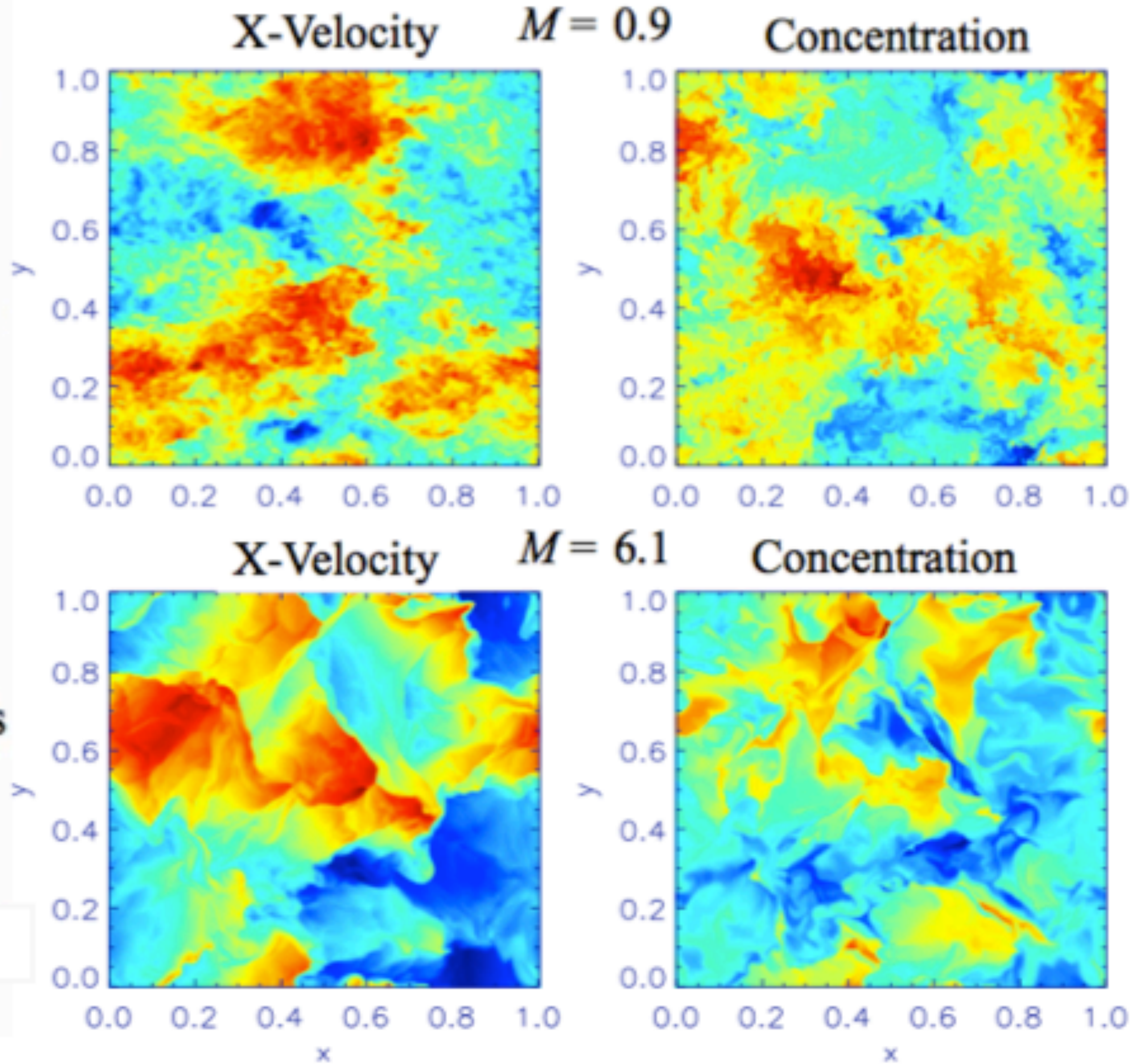
Solenoidal external force with $2\pi \le k \le 6\pi$

Isothermal equation of state

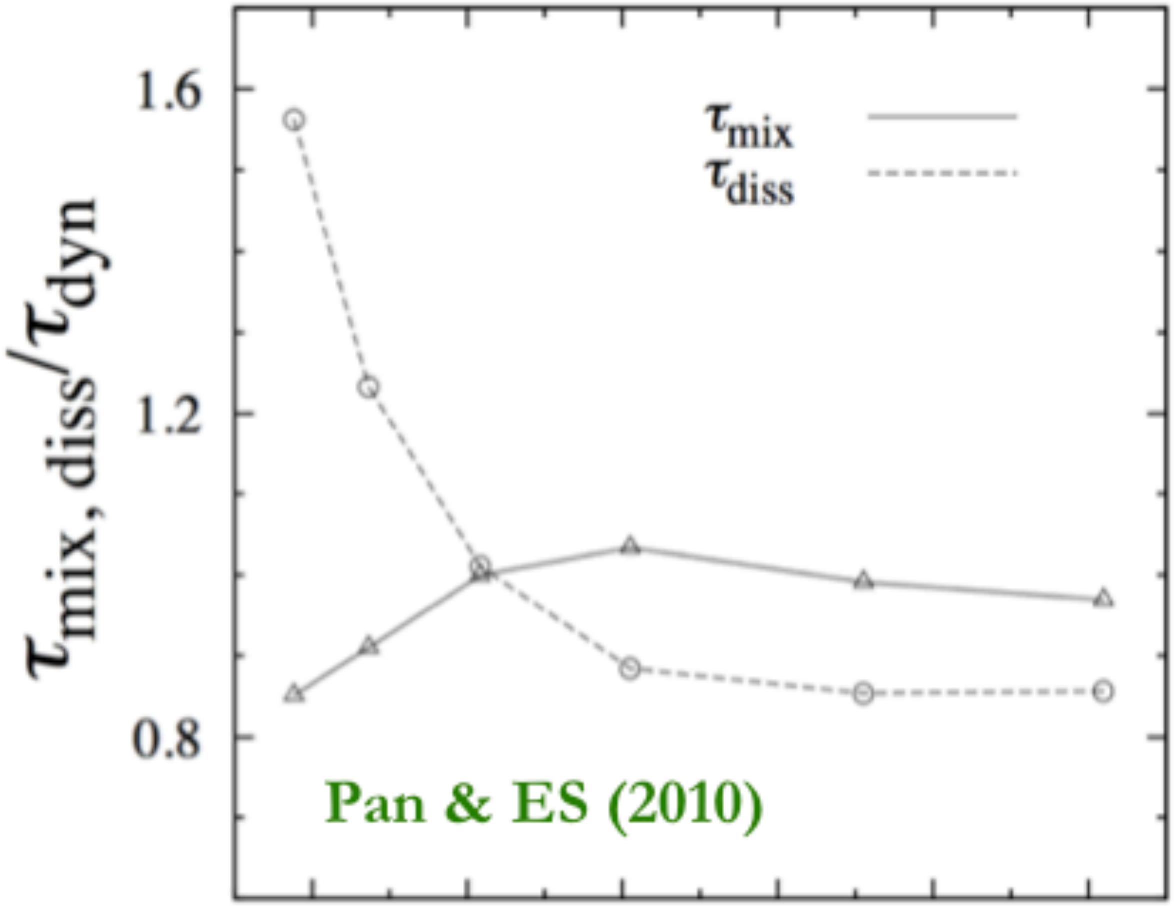
Six flows with Mach numbers $0.9 \le M \le 6.1$

The same forcing pattern for scalar sources

Pan & ES (2010)



Mixing Timescales



- Energy dissipation rate increases with Mach #

- Mixing rate decreases with M: compressible modes are less efficient at producing small scalar structures.

$$\partial_t \langle \tilde{\rho} C^2 \rangle + \partial_i \langle \tilde{\rho} C^2 v_i \rangle = 2\langle \partial_i (\tilde{\rho} \kappa C \partial_i C) \rangle \; + 2\langle \tilde{\rho} S C \rangle$$

↑ Advection term ↑ Diffusivity term ↑ Source term

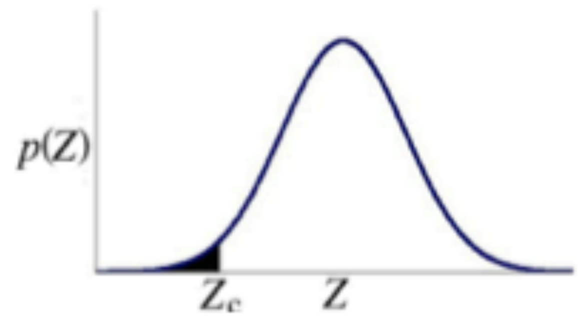
But what about the extremes?

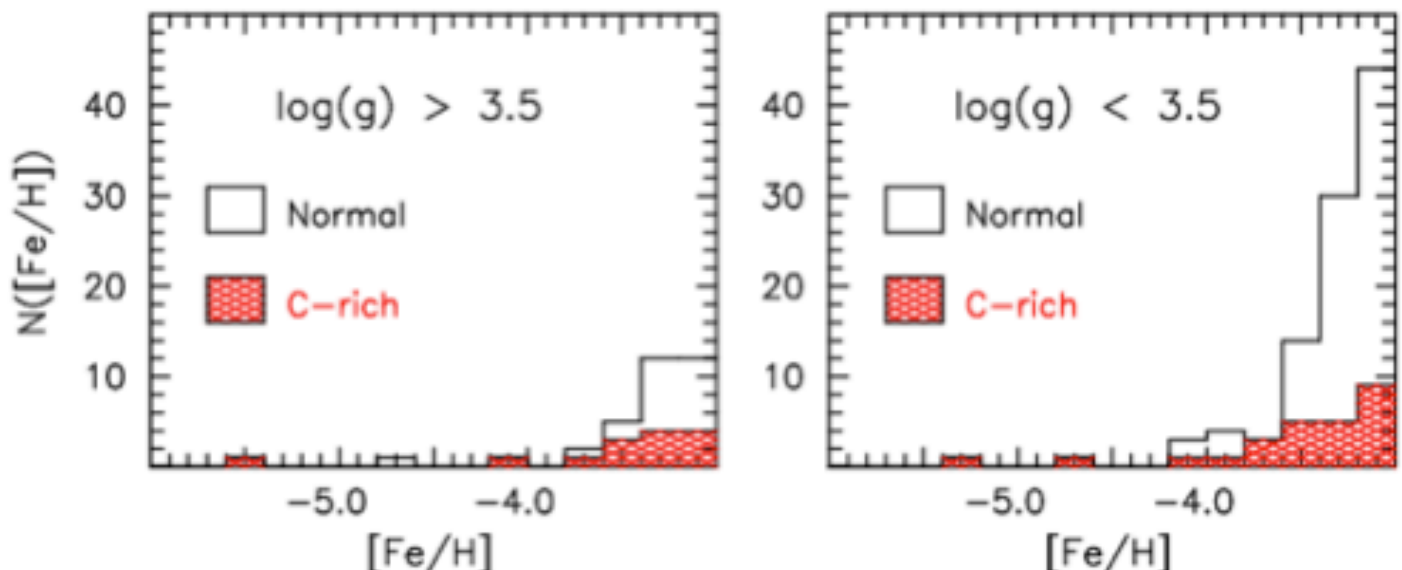
- Define a density-weighted concentration PDF,

$$p(Z;t)$$

then the mass fraction of the flow with $Z<Z_c$ ($\sim 10^{-8} - 10^{-5}$) is,

$$P(Z_c,t) = \int_0^{Z_c} p(Z';t)dZ'$$





Frebel & Norris (2013)

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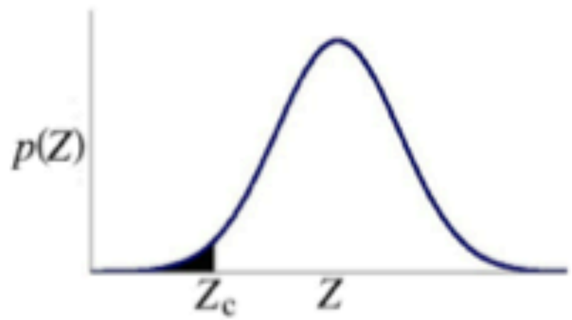

- From the concentration equation and the continuity equation, one can derive the PDF equation,

$$\frac{\partial p(Z;\mathbf{x},t)}{\partial t} + \nabla \cdot \left(p \frac{\langle \rho \mathbf{v} | C = Z \rangle}{\langle \rho | C = Z \rangle} \right) = -\frac{\partial}{\partial Z} \left(p \frac{\langle \nabla \cdot (\rho \kappa \nabla C) | C = Z \rangle}{\langle \rho | C = Z \rangle} \right) - \frac{\partial}{\partial Z} \left(p \frac{\langle \rho S | C = Z \rangle}{\langle \rho | C = Z \rangle} \right)$$

Advection term — Diffusivity term — Source term

Pan, ES, & Scalo (2012)

Closure Models

Nonlinear integral models (Curl 1963; Dopazo 1979; Janicka et al. 1979)

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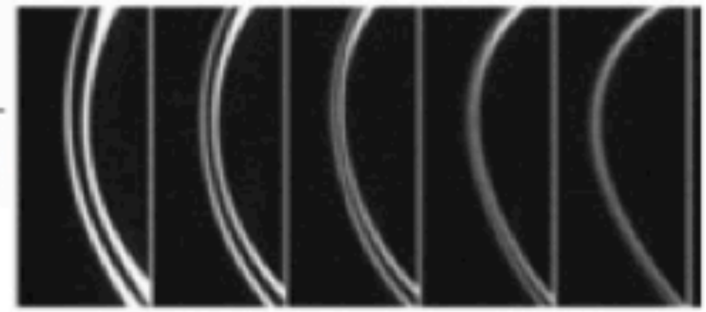
Nonlinear integral models (Curl 1963; Dopazo 1979; Janicka et al. 1979)

$$\frac{\partial p(Z;t)}{\partial t} = \gamma(t)\left\{\left[\int_0^1 dZ_1 p(Z_1;t)\int_0^1 dZ_2 p(Z_2;t)\delta\left(Z-\frac{Z_1+Z_2}{2}\right)\right] - p(Z;t)\right\}$$

Stretching ➡ Scalar sheets ➡ Mixing

$$\frac{dP(t)}{dt} = -\frac{P(1-P)}{\tau_{\rm int}} \;\Rightarrow\; P(t) = \frac{P_0}{P_0 + (1-P_0)\exp\left(\frac{t}{\tau_{\rm int}}\right)}$$



Self-convolution models (Venaille & Sommeria 2007; Villermaux & Duplat 2008)

➔ Continuous convolution model:

$$\frac{dP(t)}{dt} = \frac{P\ln(P)}{\tau_{\rm con}}$$

$$\Rightarrow\; P(t) = P_0^{\exp(t/\tau_{\rm con})}$$

➔ Generalized convolution model:

$$\frac{dP}{dt} = -\frac{n}{\tau_{\rm con}}P(1-P^{1/n})$$

$$\Rightarrow\; P(t) = \frac{P_0}{\left[P_0^{1/n} + (1-P_0^{1/n})\exp\left(t/\tau_{\rm con}\right)\right]^n}$$

Pan, ES, & Scalo (2012)

Initial Conditions

Set to be bimodal (purely polluted, and purely pristine):

$$p(Z;0) = P_0\delta(Z) + H_0\delta(Z-1),$$

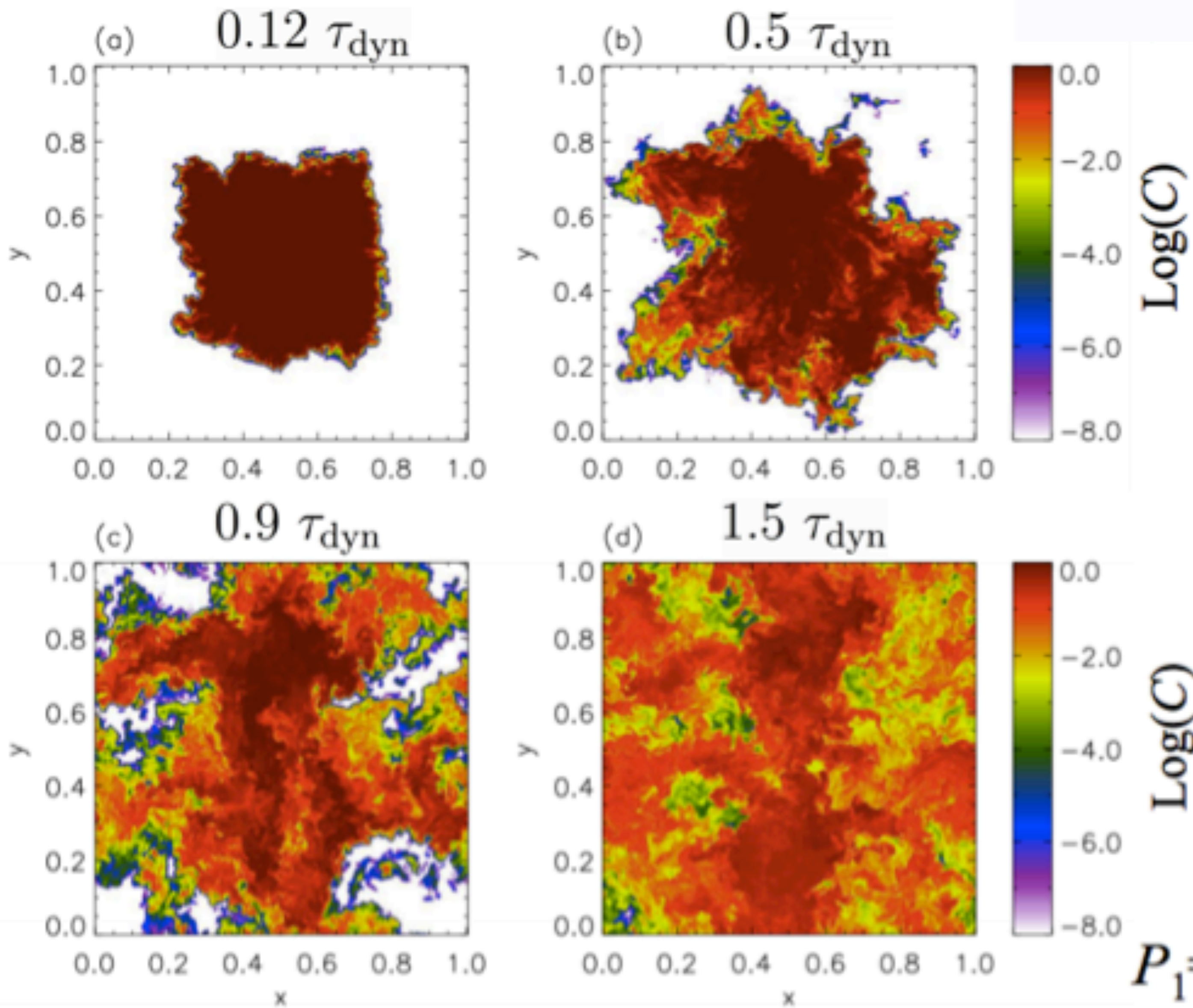
where P_0 and H_0 are the initial pristine and pollutant fractions.

Initial Pollutant region is a cube at the center of the box

Category	Pollutant Configuration	$H_0 = 0.5$	$H_0 = 0.1$	$H_0 = 10^{-2}$	$H_0 = 10^{-3}$
i	1 cube	iA	iB	iC	iD
ii	1 ball	iiA	iiB	iiC	iiD
iii	2^3 balls	iiiA	iiiB	iiiC	iiiD
iv	4^3 balls	ivA	ivB	ivC	ivD
v	8^3 balls	vA	vB	vC	vD

Pan, ES, & Scalo (2012,2013)

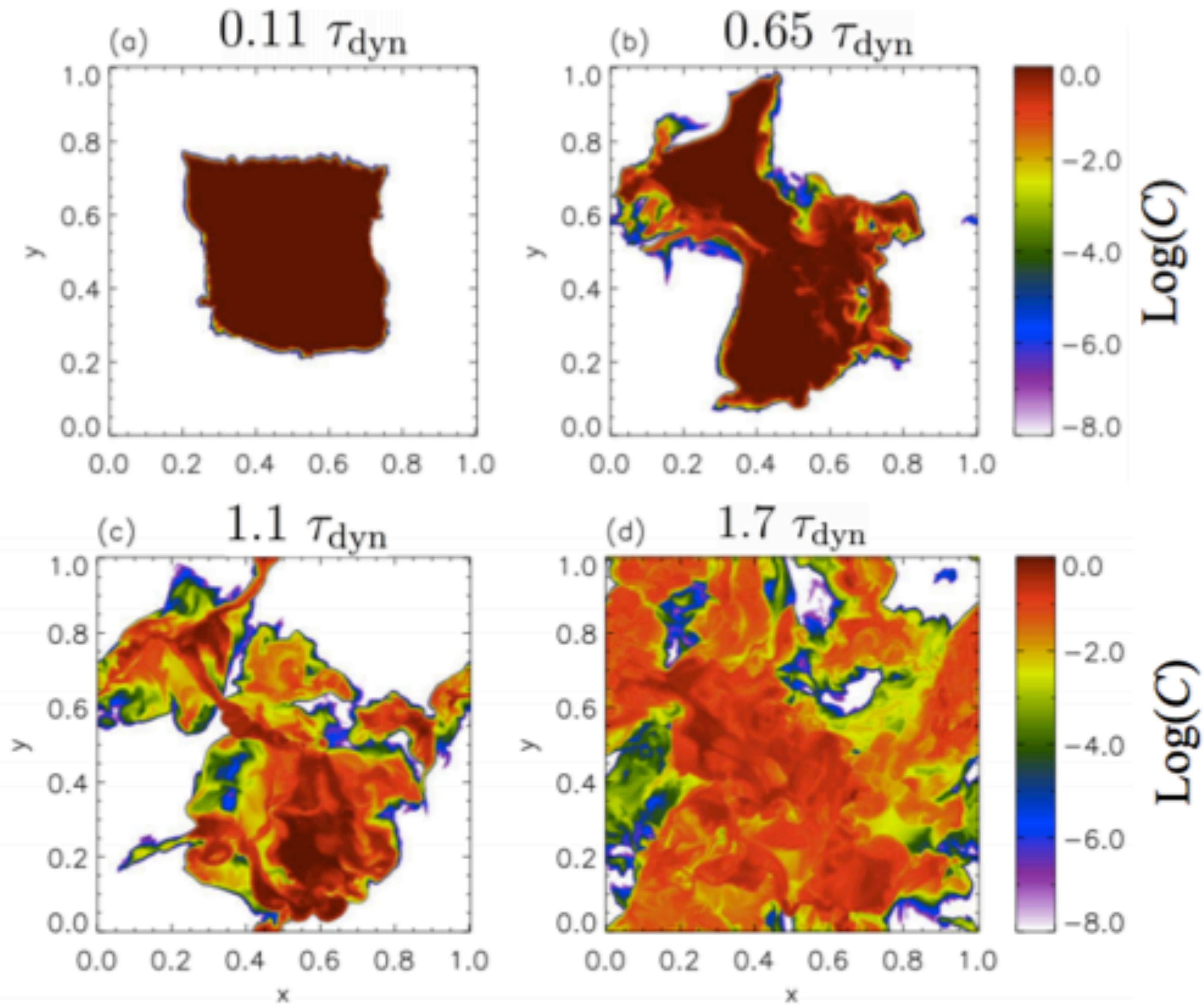
Simulation Results M=0.9



$P_1 = 0.1$

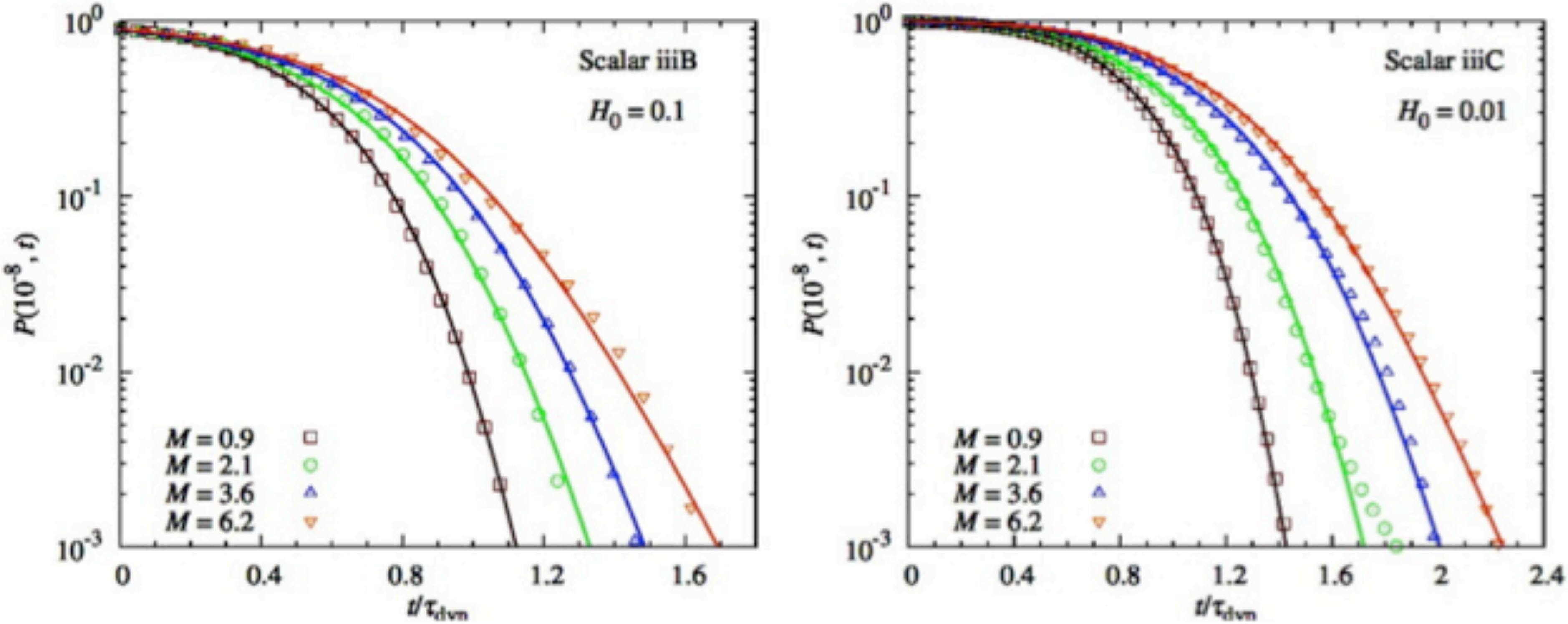
Pan, ES, & Scalo (2012,2013)

Simulation Results M=6.2



Pan, ES, & Scalo (2012,2013)

Evolution



Extremely well fit by generalized convolution model

$$\frac{dP}{dt} = -\frac{n_s}{\tau_{scon}} P(1 - P^{1/n_s})$$

Pan, ES, & Scalo (2013)

We fit it all with 2 parameters

$$\tau_{\mathrm{con1}} = [0.225 - 0.055\exp(-M^{3/2}/4)] \times \left(\frac{L_{\mathrm{p}}}{L_{\mathrm{f}}}\right)^{0.63} \left(\frac{Z_{\mathrm{c}}}{10^{-7}\langle Z\rangle}\right)^{0.015}, \quad \text{if P} \geq 0.9$$

$$\tau_{\mathrm{con2}} = [0.335 - 0.095\exp(-M^{2}/4)] \times \left(\frac{L_{\mathrm{p}}}{L_{\mathrm{f}}}\right)^{0.63} \left(\frac{Z_{\mathrm{c}}}{10^{-7}\langle Z\rangle}\right)^{0.02}, \quad \text{if P} < 0.9$$

and

$$n = 1 + 11\,\exp(-M/3.5)\left(\frac{L_{\mathrm{p}}}{L_{\mathrm{f}}}\right)^{1.3},$$

Pan, ES, & Scalo (2013)

We fit it all with 2 parameters

$$\tau_{\rm con1} = \left[0.225 - 0.055\,\exp(-M^{3/2}/4)\right] \text{ if } \mathrm{P}\geq 0.9$$

$$\tau_{\rm con2} = \left[0.335 - 0.095\,\exp(-M^{2}/4)\right] \text{ if } \mathrm{P}<0.9$$

$$n = 1 + 11\,\exp(-M/3.5)$$

$$v_T \approx \Delta x \left|\frac{d_i v_j + d_j v_i}{2}\right|$$

Smagorinsky (1963)

Pan, ES, & Scalo (2013)

Subgrid Model for Evolution Primordial Fraction

$$\frac{D(\rho P)}{Dt} = -\frac{n_s \rho}{\tau_{scon}} P(1 - P^{1/n_s}) - \dot{\rho}_{ejecta} P$$

P **-** Primordial Fraction

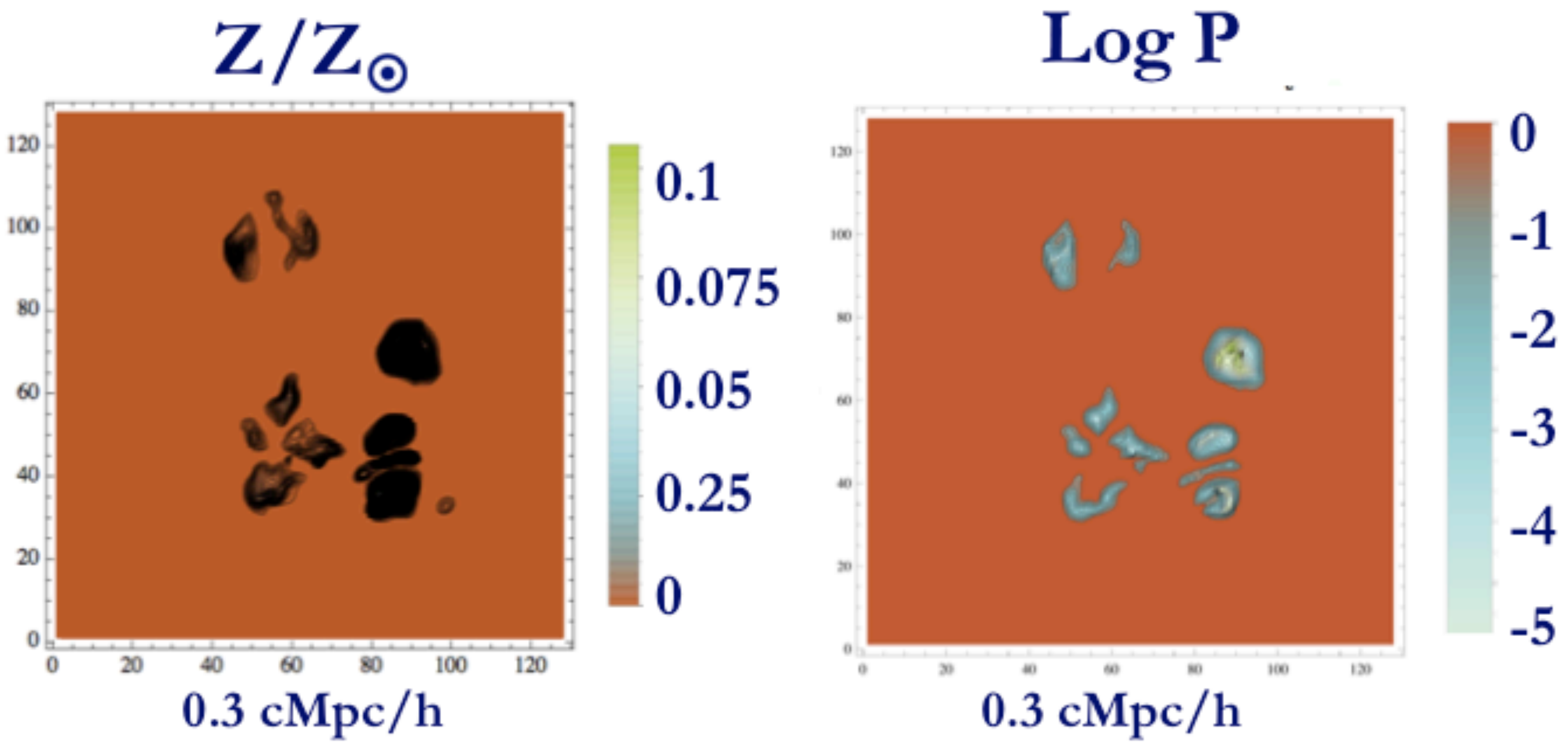
ρ – Density

$\dot{\boldsymbol{\rho}}_{ejecta}$ – Rate at which ejecta is being added to a cell

$\boldsymbol{n}_s$, $\boldsymbol{\tau}_{scon}$ – Fit functions to local turbulent Mach number and Z/Z_c provided in Pan, ES, & Scalo (2013)

Pan, ES, & Scalo (2013)

Evolution of Primordial Fraction (Test run @ z=9)



We are implementing this now into the RAMSES code.

with R. Sarmento, ES

Thanks!

If you can track metals in your simulation (or even semi-analytical model), you can track primordial fraction!